**Parametric Identification and Sensitivity Analysis combined with a Damage Model for Reinforced Concrete Structures**

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| ABSTRACT |
| The present work proposes a methodology to analyse parametric sensitivity and identify constitutive model parameters based on optimisation techniques. This issue is an important step to develop reliable constitutive models of materials with complex mechanical behaviour like concrete. In the numerical examples, we consider structures comprising reinforced concrete whose mechanical behaviour is governed by a damage model. The numerical results validate the proposed methodology and contribute to a better understanding of the damage model considered in the analyses. |
| Keywords: Optimisation, Sensitivity analysis, Damage mechanics, Reinforced concrete, Inverse Problem. |

1. INTRODUCTION

Since computers were created, advances in Computational Science as well in algorithm development have contributed to better understanding the mechanical behaviour of materials [1]. Moreover, nowadays new numerical techniques can apply analytical theories to reproduce nature phenomena. These techniques can be used to verify the mechanical behaviour of materials under different loading [2], which can be very useful considering that experimental tests can be very expensive and sometimes difficult to be performed [3]. These kinds of numerical models are denoted as effective models, because they assume hypotheses and simplifications in order to represent the physical behaviour of a system by specific mathematical relations [2]. On the other hand, the constitutive model must define parameters that are capable of qualitatively representing experimental observations [4]. In addition, the accuracy of constitutive modelling depends not only on the constitutive model, but also if its parametric identification has been performed correctly. In the context of the parametric identification of constitutive models, the use of optimisation techniques is still under explored. For example, in [5–9], the authors used the inverse technique to solve problems in applied sciences.

It is important to stress that in the present work we study the mechanical behaviour of concrete, which presents high complexity. Concrete is modelled as a composite, as it presents more than two phases in its microstructure, which have different mechanical characteristics, for example, strength and rigidity [1,10]. This material is widely used in Civil Engineering structures [11], whereby the concrete comprises three different phases: i) coarse aggregates ii) mortar iii) interface zone, surrounding the inclusions. Each one of these phases directly affects the mechanical behaviour of the material and, therefore, they must be properly modelled [12].

Concrete presents a quasi-brittle mechanical behaviour [13,14], where irreversible strains as well as progressive loss of rigidity can be observed during its fracture process [15–17]. One important particularity of this material is that it does not present symmetric behaviour in tension and compression [18]. Different physical-mathematical models have been used to simulate this non symmetric behaviour, especially models based on Damage Mechanics [19].

In the models based on Damage Mechanics, the damage process of the material is represented by thermodynamic state variables which can be scalars or defined by tensors. The global behaviour is defined by the evolution of these variables in time, where for each time “t” the equilibrium of the irreversible thermodynamic problem must be obtained, in agreement with the first and the second laws of thermodynamics [20]. The damage models have been used satisfactorily to simulate different kind of materials, for example, biological materials, metals [17,21–24] and composites [25]. Concerning concrete applications, various studies have been conducted [13,15,19,26–39].

In the present work, we propose a methodology to analyse parametric sensitivity and identify constitutive model parameters based on optimisation techniques. As the correct parametric identification is particularly important to accurately represent material behaviour, new techniques must be developed to make the efficient use of robust models possible. The viability of the proposed methodology will be verified by performing numerical analyses of structures where either the Finite Element Method or the Boundary Element Method can be used. In the last 30 years, Continuum Damage Mechanics (CDM) has experienced a great development concerning damage constitutive models to concrete specifically when dealing with anisotropic mechanical behaviour [CINCO ARTIGOS]. However, the methodology will be adopted to identify the parameters of the damage model proposed by Mazars & Pijaudier-Cabot [40]. The Mazars’ model is a reference model in the CDM and it has already proved its reliability in many situations when applied in numerical analysis of concrete structure.

1. METHODOLOGY FOR PARAMETRIC IDENTIFICATION OF THE DAMAGE MODEL

In this section, we present the proposed methodology to determine the parameters of the damage model described in Mazars & Pijaudier-Cabot [40].

Mazars’ Model

This model has been proposed by Mazars [1] and the damage is represented by the scalar variable D (with 0 ≤ D ≤ 1) whose evolution occurs when the equivalent extension deformation is bigger than a reference value. The plastic deformations evidenced experimentally are not considered. The equivalent extension deformation is given by:

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|  | (1) |

where  is a principal deformation component, being its positive part, i. e.: .

The damage activation occurs when εd0, being εd0 the deformation referred to the maximum stress of an uniaxial tension test. Thus the criterion is given by:

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| *with S(0) = εd0* | (2) |

Considering the thermodynamics principles, the damage evolution can be expressed by:

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| *if f < 0 or f = 0 and < 0* | (3a) |
| *if f = 0 and* | (3b) |

where , i. e., D time derivative;  is written in terms of  and defined continuous and positive.

As the concrete behaves differently in tension and compression, the damage variable D is obtained by combining properly the variables DT and DC, related to tension and compression, respectively, as follows:

|  |  |
| --- | --- |
| *where* | (4) |

where DT and DC are given by:

|  |  |
| --- | --- |
|  | (5a) |
|  | (5b) |

In Eqs (5) AT and BT are parameters related to uniaxial tension tests while AC and BC are obtained from uniaxial compression tests. To compute the αT and αC values defined in Eq.(4), we have to obtain, initially, the deformations εT and εC associated, respectively, to tension and compression states as follows:

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| --- | --- |
|  | (6a) |
|  | (6b) |

where **I** is the identity tensor, E the elastic modulus of a non-damaged material, and are, respectively, positive and negative parts of the stress tensor obtained from the relation , where  is the elastic fourth order tensor of the non-damaged material.

Thus the coefficients  and  are obtained by the following expression:

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|  | (7a) |
|  | (7b) |

where and are, respectively, positive and negative parts of the deformations εT and εC defined in Eq. (6);  is given by: .

Finally, the constitutive relation can be expressed in terms of the actual deformation tensor as follows:

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|  | (8) |

Sensitivity analysis

Initially an analysis of parametric sensitivity of the damage model must be performed, which consists of quantifying the variations of the output results when the input values are modified [41,42]. This technique is important for developing physical-mathematical models [43] in several fields of Sciences [43–45].

To make the sensitivity analysis, we considered the One-Factor-at-a-Time or the One-at-a-Time approach, which consists of independently varying the input values, one at a time, maintaining the other values constant, in agreement with a pre-stablished standard scheme [46]. To do this, the test function must be differentiated [47] as proposed in Eq (1), where is the normalised sensitivity function; is the n-dimension variable defined in the problem; and is the test function that represents the physical-mathematical model.

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|  | (1) |

In the case of the present work, the function to be analysed is the stress function used to describe the concrete behaviour under uniaxial loads cases, according to Eq. (2).

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|  | (2) |

The scalar damage variable () is given by Eq. (7), where the damage evolution law for uniaxial tension is defined by Eq. (3), while Eq. (5) represents the damage evolution law for uniaxial loads in compression. Variable represents the strain from which the system presents damage, is the equivalent strain necessary to verify the damage process, which for uniaxial tension is defined by Eq. (4) and for uniaxial compression is given by Eq. (6).

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| --- | --- |
|  | (3) |
|  | (4) |
|  | (5) |
|  | (6) |
|  | (7) |

The and values represent the contributing plots in the scalar damage variable () for the effects of tension and compression, respectively. Observe that for uniaxial tension, we have and while for uniaxial compression we have On the other hand, for other cases, we have non null values for and , where + .

The model parameters are defined by: and (see Eqs. (3) and (5)). We intend in this section to study the influence of each one of these parameters on the value of the scalar damage variables ( and ).

In Eq. (6), represents the Poisson’s ratio for the concrete, which in the present work has been adopted to be equal to 0.20, while represents the initial Young’s modulus for the concrete. For the sensitivity analyses discussed in Section (3.1), we considered the average strength in compression () equal to 30 MPa. Then, the value has been obtained by Eq. (8) and the average strength in tension computed by Eq. (9), which enables us to obtain the limit elastic strain by using Eq. (10) (the same procedure is adopted in [48,49]).

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| --- | --- |
|  | (8) |
| M | (9) |
|  | (10) |

To verify the influence of the parameter values on the damage modelling, we considered a disturbance rate equal to 50% in their values. On the other hand, their limit values were defined according to Mazars’ work [50], where: ; ; and .

Table 1 defines the standard scheme adopted for the study of parametric sensitivity of each parameter in terms of the applied load and in agreement with the limit values defined by Mazars [50]. To obtain the stress versus strain curves, we applied a maximum uniaxial strain in tension equal to 0.060% while for uniaxial compression cases, we considered the maximum uniaxial strain equal to 0.60%.

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| Table 1 - Values of constitutive parameters ( and ) for the sensitivity analysis. |
| |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | | **Analysis** |  | **(MPa-1)** | **Load** | **Strain (%)** | **Maximum strain (%)** | |  | 0.70 | 10000.00 | Uniaxial tension | 0.0127 | 0.060 | |  | 0.85 | 10000.00 | 0.0127 | 0.060 | |  | 0.70 | 55000.00 | 0.0127 | 0.060 | |  | 1.00 | 1000.00 | Uniaxial compression | 0.0127 | 0.60 | |  | 1.25 | 1000.00 | 0.0127 | 0.60 | |  | 1.00 | 1500.00 | 0.0127 | 0.60 | |

* 1. The inverse analysis to compute the constitutive parameters

To obtain the constitutive parameters, we adopt the technique of inverse problems based on optimisation concepts, which consists of obtaining the causes from the observed effects. In the present work, the variables of the Mazars & Pijaudier-Cabot model are assumed as the causes of the problem. As a result, we obtain the complete curve of stress versus strain for the uniaxial cases of compression and tension.

The inverse technique applied to compute the model parameters can be understood as an adjustment problem. This technique has been widely used due to its versatility, and is applied for different kind of problems, for example, in solids mechanics [51–53], thermodynamics [54] and soils mechanics [55, 56]. In the present work, the optimisation technique is used to minimise the deviation between the numerical results and the results observed experimentally, where the Objective Function () is given by Eq. (11) (this function has also been used [57, 58]).

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| --- | --- |
|  | (11) |
|  | (12) |

In Eq. (11) represents the experimental or observed stresses; represents the numerical stresses obtained by the damage model considering a set of variables denoted as ; represents the number of points related to the experimental results; are the project variables of the problem, where and , respectively, the inferior and superior limits of these variables.

We adopted the “Artificial Bee Colony (ABC)” as the optimiser algorithm for the inverse problem (see Figure 1). This algorithm was developed in [59] and it is based on bee behaviour in the search for food. This algorithm is characterized as a swarm-type computational intelligence, in which the solution space is explored by a population of particles that share information with each other. The ABC optimiser algorithm is shown in Figure 1 and further details on the optimiser algorithm can be found in [59–62].

The algorithm has two stages. In the first stage, the input values must be defined, for example: the total population of bees (); the number of employed bees (); the number of onlooker bees (); the trial limit for activating the scout bees, defined according to Eq. (13); the problem dimension (); number of iterations (); the initial swarm population.

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|  | (13) |

The second stage of the algorithm consists of the iterative procedure which is divided into three sub-stages: ) Searching for the employed bees, ) searching for the onlooker bees, ) searching for the scout bees.

When the search for the employed bees is performed, solutions are obtained from Eq. (14), where the value is scalar and defined randomly between the limits [-1, +1]. In Eq. (14), the value corresponds to an array vector which can vary from 1 to . The subscripts and are defined randomly to improve the solution where [1, 2, ..., ], and [1, 2, ..., ]. However, must be different from .

|  |  |
| --- | --- |
|  | (14) |

Then, solution is defined as the new solution, considering the greedy selection criteria where the fitness solution () is obtained from Eq. (15), where is the value of the objective function for particle .

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| --- | --- |
|  | (15) |

The next sub-stage is the search for the onlooker bees which will improve the selected solutions locally by considering the roulette selection process which is based on the probability of particle selection as defined in Eq. (16). Therefore, this sub-stage aims to improve the solutions obtained in the previous sub-stage related to the search of employed bees.

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|  | (16) |

The solution adopted for the improvement is chosen randomly by considering the roulette selection process. Then, Eq. (14), as well as the greedy selection criteria, must be verified for all movements.

When the iterative procedure related to the search for the onlooker bees is finished, we store the best solution and the third sub-stage, related to the search for the scout bees, begins. In this sub-stage, we replace only one particle that has exceeded the limit defined in Eq. (13). Observe that the trial variable (defined in Figure 1) stores the number of solutions that have not been improved according to the greedy selection criteria. Therefore, when a solution is not improved in the previous sub-stage, if the trial variable is bigger than the established limit, it will be replaced by Eq. (17):

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|  | (17) |

Where and correspond to the range of possible solutions and is a vector of random numbers which can vary from 0 to 1.

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| Figure 1 – Standard ABC. |

In the present work, for the Bee Colony Algorithm, we adopted the number of swarm population according to [59], where there are 50% of employed bees and 50% of onlooker bees. Besides, the number of iterations was equal to 250 for each repetition.

For the tests related to the parametric identification, the optimisation routine was executed 1000 times with a fixed population equal to 48 individuals (24 employed bees and 24 onlooker bees). Observe that we performed initial tests to verify that this number of populations presents satisfactory results in terms of equivalence of the numerical model. The coupling between the damage model ant the optimisation algorithm is presented in Figure 2.

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| Figure 2 - Coupling between the numerical model and the optimisation process. |

Data found in the literature are adopted, according to Pituba and Fernandes [32]. The reference curves in the uniaxial compression and uniaxial tension are presented in Figure 3. The stress-strain curve shown in Figure 3b is an experimental result in a compression test carried out by Àlvares [68]. On the other hand, Figure 3a shows the expected response of this kind of concrete according to the parameters obtained by Àlvares [68], Pituba and Fernandes [32] using Mazars’ model.

The mechanical characteristics of the concrete are defined in Table 2. The and values correspond to the peak stress in compression and in tension, respectively, while and are the strains related to these peak stresses. Therefore, and are only values that help in the classification by strength of concrete according to design standard, in which case the concrete is classified as C30.

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| Figure 3 - Stress versus strain curves: (a) in tension e (b) in compression [32]. |

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| Table 2 - Mechanical properties of the stress versus strain curves to use the optimisation algorithm. |
| |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | | **Concrete** | **(MPa)** | **(MPa)** | **(GPa)** | **υ** | **(%)** | | C30 | 30.00 | 2.29 | 29.20 | 0.20 | 0.0070 | |

The parametric identification will be made for this concrete to verify the damage variables in uniaxial tension and uniaxial compression. Observe that the variable is defined from the experimental test of uniaxial tension, as commented previously.

1. RESULTS AND DISCUSSION

In this section, the sensitivity analysis and parametric identification results are presented, as well as the numerical analysis of structures.

* 1. Sensitivity Analysis for the Mazars & Pijaudier-Cabot Damage Model

For the analysis performed in this section, we adopted the variation of the constitutive parameters defined in Table 1 of section 2.1. Figure 4 presents the results for the sensitivity function , related to the damage variables in uniaxial tension. This figure also presents the normalised behaviour related to the stress versus strain curve.

In Figure 4, we can observe that both variables, and , influence the stress versus strain curve as they modify its non-linear part. Observe that for strains smaller than , these variables have no influence because they refer to the elastic part of the curve, with no damage. Besides, for the initial stages of softening, the influence of variable is bigger than variable , but from the strain equal to 3.10-4, variable presents the biggest influence.

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| Figure 4 - Sensitivity Analysis of Variables and . |

In Figure 5, where the analyses and are presented, we can observe that the variation of the variable in the analysis increased the loss of rigidity for the initial stages of non-linear strains, which is in agreement with the sensitivity analysis presented in Figure 4. Besides, for uniaxial tension, the and values adopted did not present any hardening, which is also in agreement with the sensitivity analysis, as no sensitivity value () was positive (see Figure 4). As the analysis is based on derivatives, the slope at the beginning of the non-linear part of the curve will have concavity facing upwards, which represents the softening phenomenon of the system.

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| Figure 5 - Stress versus strain curve, considering variations in the damage parameters for uniaxial tension. |

The same procedure was adopted for the uniaxial compression case to verify the influence of the and variables in the stress versus strain curve of the material (see Figure 5).

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| Figure 6 - Sensitivity Analysis of Variables and . |

In Figure 6, we can observe that the damage and its variables only have influence on the curve after the strain is equal to 0.045%. In terms of sensitivity analysis, we can say that both and variables can affect the non-linear part of the curve. The variable has influence aall stages of the curve while the variable has a bigger influence for stress values next to the peak stress. Therefore, we can conclude that variations in the value can greatly influence the value of the peak stress of the material.

In order to verify the qualitative influence of the parameters and , Figure 7 presents the results for the analysess to . In this case, we can conclude that an increase in the value of the parameter increases the value of the peak stress of the material. Observe that for the analysis, the peak stress is equal to -40.89 MPa while for the analysis, the peak stress is -48.01 MPa, which corresponds to an increase of 17.41%. For the simulation, the peak stress is decreased in compression, i.e., the concrete behaviour is weakened, and is equal to -29.05 MPa, which refers to a reduction of 28.96%. This is in agreement with the qualitative analysis presented in Figure 6, i.e., the variation of the parameter greatly influences the response of the material in compression.

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| Figure 7 - Stress-strain curves with different damage parameters for uniaxial compression. |

We have also performed parametric simulations considering the bending problem in concrete plates. To do this, we used a numerical model based on the Boundary Element Method (BEM) (for more details, see [63–65]). The concrete plate considered in this section was also analysed in Fernandes *et al.* [66]. The square plate, whose side measures 100cm, is simply supported with a uniform load equal to 0.40 kN/cm2 applied over all its domain. The plate has a thickness equal to 4 cm and for its discretisation, we considered 24 elements over the external boundary and 72 cells over the domain, where the inelastic stresses are approximated (see Figure 8). The concrete properties are defined in Table 1.

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| Figure 8 - Geometry and discretisation for the simply supported plate. |

The damage parameters are defined in Table 3 and Figure 9 presents the load versus displacement curve for the central point of the plate while Figure 9b presents details of the different responses for the loads where the damage process is intense.

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| Table 3 - Parameter Values of the Mazars & Pijaudier-Cabot model [40] adopted for the analyses. |
| |  |  |  |  |  | | --- | --- | --- | --- | --- | | **Simulation** |  |  |  |  | | BEM1 | 0.70 | 10000.00 | 1.00 | 1000.00 | | BEM 2 | 0.85 | 10000.00 | 1.00 | 1000.00 | | BEM 3 | 0.70 | 55000.00 | 1.00 | 1000.00 | | BEM 4 | 0.70 | 10000.00 | 1.25 | 1000.00 | | BEM 5 | 0.70 | 10000.00 | 1.00 | 1500.00 | |

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|  | **((a)** |
|  | **(b)** |
| Figure 9 - Load versus displacement curve for the central point of the plate. (a) the complete curve; and (b) Responses for displacement bigger than 5.50 mm. | |

Comparing the results related to the BEM4 analysis to the BEM1 and BEM5 analyses, it can be said that the compression parameter did not significantly influence the structure rigidity. However, if we compare the BEM1 analysis to the responses related to BEM2 and BEM3, we can observe that variations in the and parameters modify the plate rigidity, which is in agreement with the sensitivity analysis shown in Figure 4 and Figure 5. An increase in the and variables implied a reduction in rigidity, and the variable caused the most significant change in the BEM3 simulation.

The plate behaviour discussed above can be explained by the fact that the plate is not reinforced, and therefore its strength in tension is much smaller than its strength in compression (the strength in tension is 10% of the strength in compression [67]). Thus, in this case, the parameters related to the uniaxial tension are very important.

* 1. Analyses of reinforced concrete structures by using the proposed methodology

In this section, reinforced concrete structures are analysed by using a numerical model based on the Finite Element Method (see more details in Pituba & Fernandes [32]). In this model, the cross section of the beam element is discretised into layers (see Figure 10) where the distortional strain is not taken into account. To verify the numerical results, we consider the experimental results obtained in [68]. The mechanical behaviour of the concrete is governed by the Mazars & Pijaudier-Cabot [40] damage model, while a perfect plastic model governs the mechanical behaviour of the steel (see [19,69]). The material properties related to the reinforcement are defined in Table 1 and they are in agreement with [19,32]. On the other hand, the concrete property parameters are defined by the inverse problem, i.e., from the parametric identification, where optimisation techniques are applied (see section 3.2.1).

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| Figure 10 - Finite Element Model with stratified cross section. |

The beam analysed in this section has two concentrate loads, as depicted in Figure 11, where the cross sections are also defined. In Figure 11, we can observe three different compositions for the reinforcement (see more details in Pituba & Fernandes [32]). In the numerical model, we defined 20 finite elements, whereby the cross sections are divided into 15 layers.

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| Figure 11 - Geometry, loads and cross-sections for the beam [68]. | |

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| Table 4 - Parameters values for the steel CA-50 used in the reinforcement [19,32]. |
| |  |  |  | | --- | --- | --- | | **Material Properties** | **Value** | **Unit** | | Young’s modulus | 196 | GPa | | Yield Stress (for cross section with 3 | 500.00 | MPa | | Yield Stress (for cross section with 5 e 7 | 420.00 | MPa | | Mass density | 7.850.00 | kg/m³ | | Rupture strain | 1.086 | % | |

In order to verify the representativeness of the numerical responses provided by the damage model in relation to the experimental responses in uniaxial stress tests when performing parametric identification, Willmontt and Pearson tests were performed according to equations (18) and (19).

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|  | (18) |
|  | (19) |
|  | (20) |

To make the qualitative analysis, we considered the proposal defined in the study carried out by Camargo and Centelhas [70], which is detailed in Table 5.

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| Table 5 – Classification of confidence index [70]. |
| |  |  | | --- | --- | | **Value** | **Performance** | | > 0.85 | Excellent | | 0.75 0.85 | Very good | | 0.65 0.75 | Good | | 0.60 0.65 | Medium | | 0.50 0.60 | Regular | | 0.40 0.50 | Bad | | 0.40 | Terrible | |

3.2.1 Parametric Identification for the Damage Model by using Computational Intelligence

For the concrete used in the work [32] and defined in section 2.1, Figure 3 and Table 2, we used the optimisation criteria by considering an inverse procedure. The limits of design variables, which in the present work are the damage variables, are: ; ; and .

The results for the uniaxial tension are presented in Figure 12. We obtained the values 0.9831 and 7985.25, respectively, for the and parameters, where the OF is equal to 0.6367. In Figure 12, we obtained a performance rate equal to 0.9979, a coefficient equal to 0.9930 and a coefficient equal to 0.9944. It can be observed in Figure 13 that the convergence was achieved after 50 iterations of the optimisation algorithm.

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| Figure 12 - Parametric identification for the stress versus strain curve considering uniaxial tension. |
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| Figure 13 - Convergence analysis of the optimisation algorithm for uniaxial tension. |

In the case of uniaxial compression (see Figure 14), the value obtained for the OF is 1.6020 and the values computed for the project variables are: = 0.7003 and = 935.51. Moreover, the values obtained in the performance analysis are: (a) = 0.9886; (b) = 0.9589; and (c) = 0.9681. In this case, the convergence of the optimisation algorithm was obtained before 50 iterations (see Figure 15).

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| Figure 14 - Parametric identification for the stress versus strain curve considering uniaxial compression. |
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| Figure 15 - Convergence analysis of the optimisation algorithm for uniaxial compression. |

3.2.2 Numerical analyses of reinforced concrete beams

In this section, we present the results related to the reinforced concrete beams by adopting the same values obtained by the solution of the inverse problem for the damage model parameters detailed in Section 3.2.1 and whose values are shown in Table 6. This table also presents the parameter values obtained by Pituba and Fernandes [32], where the square minimum technique was used to compute them. In Table 6, we can observe more difference for the compression values, where 17.60% and 10.90%, respectively, are the differences for the and parameters.

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| Table 6 – Variables of the damage model for the class C30 concrete. |
| |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | | **Concrete** | **(GPa)** | **(%)** |  |  | **(MPa-1)** | **(MPa-1)** |  | | Present work | 29.20 | 0.0700 | 0.9831 | 0.7003 | 7985.25 | 935.51 | 0.20 | | Pituba & Fernandes [32] | 29.20 | 0.0700 | 0.9950 | 0.8500 | 8000.00 | 1050.00 | |

The load versus displacement curves related to the beams defined in Figure 11 are depicted in Figure 16, Figure 17 and Figure 18. It can be observed that the damage model was able to represent the loss of rigidity observed in the experimental tests (see Figure 16 and Figure 18). Therefore, it can be concluded that the numerical model can represent the non-linear phenomenon which occurs mainly due to the fracture process in the concrete. Besides, it can also be observed that the system remains elastic up to the displacement equal to 1 mm when the damage is almost null () and the adoption of different values for the and parameter did not significantly modify the load versus displacement curve. However, for loads bigger than 40% of the limit load, the beam presents a high value of the damage variable D and therefore, the and parameters have more influence on the mechanical behaviour of the beam. In all the analyses, the limit load is achieved when the stress in the longitudinal reinforcement is equal to the yield stress of the steel.

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| Figure 16 - Load versus displacement curve for the beam with 3ϕ10 mm. |
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| Figure 17 - Load versus displacement curve for the beam with 5ϕ10 mm. |

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| Figure 18 - Load versus displacement curve for the beam with 7ϕ10 mm. |

In Table 7, we present the dispersion of the numerical results compared to the experimental results, where refers to the Mean Squared Error and refers to the Root Mean Square Error. Observe that the computes the error in its own measure unit of the variable [71]. It can be observed in Table 7 that all numerical simulations presented satisfactory correlations with the experimental average.

In Figure 16, Figure 17 and Figure 18, it can be observed that the load versus displacement curves obtained with the proposed methodology are similar to the ones presented in [32]. Besides, this similarity can also be observed by analysing Table 7, which validates the proposed methodology for parametric identification. However, it is important to stress that the proposed methodology is automatic, efficient, robust, and presents a low computational cost and can define the parameter values much faster than the methodology used in [32], which was not automatic. Therefore, user intervention is required. Thus, it can be concluded that to make the parametric identification of a constitutive model, the present methodology is a good alternative to the methodology used in [32].

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| Table 7 – Correlation between the numerical models and the experimental tests of the beams. |
| |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | | **Experimental** | **Numerical Model** |  |  |  |  | | **Beam 3** | Pituba and Fernandes | 6.9533 | 2.6369 | 0.9831 | 0.9769 | | Present work | 8.5357 | 2.9216 | 0.9789 | 0.9719 | | **Beam 5** | Pituba and Fernandes | 30.6360 | 5.5350 | 0.9635 | 0.9578 | | Present work | 36.5297 | 6.0440 | 0.9554 | 0.9496 | | **Beam 7** | Pituba and Fernandes | 28.9784 | 5.3832 | 0.9709 | 0.9657 | | Present work | 37.1036 | 6.0913 | 0.9621 | 0.9584 | |

1. CONCLUSIONS

In the present work, we presented a methodology based on optimisation techniques for parametric identification of constitutive models applied to materials with complex mechanical behaviour due to their heterogeneous microstructure, such as concrete. To do this, we applied the methodology to a well-known constitutive model in the literature.

The damage models are widely used in different fields of science, as well as in Civil Engineering, where these models are mainly considered for concrete structures. However, the parametric identification requires still more development, because efficient and robust methodologies are necessary to obtain constitutive models that accurately represent the response observed experimentally. It is also important to stress the importance of having well elaborated parametric sensitivity and parametric identification analyses because these analyses can obtain good correlations with the experimental responses when the constitutive model is used.

In the present work, we developed a methodology for parametric identification where a bio-inspired optimisation model was used, which is a stochastic technique. The application of this kind of technique to solve the parametric identification problem is not very used, but it can be very interesting for complex simulations involving a bigger number of variables and for different constitutive models.

The sensitivity analysis showed how the model parameters have influenced the material response when the constitutive model is applied to govern the material behaviour. Therefore, we can predict which parameter must be modified to verify the global response of the structure. On the other hand, the present work has validated the proposed methodology by analysing some numerical examples, where concrete plates as well as reinforce concrete beams were considered. The proposed methodology has shown to be robust and efficient and is able to compute the model parameters without user intervention. In future works, we intend to apply this methodology in parametric identification of models with complex formulation as anisotropic damage models, as instance.

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